

Supersymmetric $SO(10)$ Simplified¹

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Abstract

In the context of supersymmetric $SO(10)$ grand unified models, it is shown that the gauge symmetry breaking as well as a natural doublet–triplet splitting can be achieved with a minimal Higgs system consisting of a single adjoint and a pair of vector and spinor multiplets. Such a Higgs spectrum has been shown to arise in the free fermionic formulation of superstrings. Since the symmetry breaking mechanism relies on non–renormalizable operators, some of the Higgs particles of the model turn out to have masses somewhat below the GUT scale. As a consequence, the unification scale is raised to about $2 \times 10^{17} \text{ GeV}$ and $\sin^2 \theta_W$ is predicted to be slightly larger than the minimal SUSY– $SU(5)$ value. Including threshold uncertainties, which turn out to be surprisingly small in the model, we show that $\sin^2 \theta_W$ prediction is consistent with experiments.

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I. Introduction

There has been a rebirth of interest in supersymmetric grand unification since the improved measurements of the low energy gauge couplings has confirmed that supersymmetry leads to an astonishingly accurate unification of couplings [1]. The minimal SUSY-GUT prediction for $\sin^2 \theta_W$ is 0.2334 ± 0.0036 to be compared with the experimental value of 0.2324 ± 0.0003 [2]. (We have combined the experimental and theoretical uncertainties in quadrature.) It has long been believed by many theorists that low energy supersymmetry is a necessary ingredient of grand unification anyway if the gauge hierarchy problem is to be solved in a satisfactory manner.

One aspect of the gauge hierarchy problem is the issue of “doublet triplet splitting”, ie., keeping the pair of Higgs doublets of the supersymmetric standard model light while giving their color triplet partners superheavy masses to avoid excessive Higgs or Higgsino-mediated proton decay. There are several mechanisms that have been proposed for achieving doublet-triplet splitting in SUSY-GUTs without fine-tuning of parameters: the “sliding singlet mechanism” [3], the “missing partner mechanism” [4], the “Dimopoulos-Wilczek mechanism” [5] and the “pseudo-goldstone-boson mechanism” [6]. There is also the option of fine-tuning parameters of the superpotential to achieve doublet-triplet splitting. The non-renormalization theorem of supersymmetry will make such a fine-tuning stable under radiative corrections. Apart from being aesthetically unappealing, it seems unlikely that the small numbers required for this procedure ($\sim 10^{-14}$) would arise in a more fundamental theory such as the superstring theory.

In SUSY $SU(5)$ the only way to do doublet triplet splitting naturally is the missing partner mechanism [4], which requires the existence of Higgs multiplets in the representations **50**, $\overline{\mathbf{50}}$, and **75**. Aside from the lack of economy involved in the introduction of these rank-four tensors, it is questionable whether such high rank representations would be allowed if the SUSY-GUT arises from an underlying superstring theory.

$SO(10)$ is in a number of ways a more attractive group for grand unification. All the particles of a family are unified into a single irreducible representation, the right-handed neutrino automatically emerges, violation of $(B - L)$ allows the generation of a sphaleron-proof cosmological baryon asymmetry, and anomaly cancellation is automatic in $SO(10)$, among other things [7].

In $SO(10)$ it appears that the only possibility for natural doublet-triplet splitting is the Dimopoulos-Wilczek mechanism [5]. This mechanism is quite simple. In its simplest form the masses of the colored Higgs(ino) fields arise from a term $T_1^a A^{ab} T_2^b$ (where the T_i are **10**'s of $SO(10)$) with the vacuum expectation value of the adjoint Higgs, A^{ab} , being in the direction $\langle A \rangle = \text{diag}(0, 0, a, a, a) \otimes (i\tau_2) \propto (B - L)$. This form of the vacuum expectation value, which we call the Dimopoulos-Wilczek (DW) form, gives a Dirac mass assumed to be of order the GUT scale to the color **3** + $\overline{\mathbf{3}}$ Higgsinos and Higgs, $\overline{\mathbf{3}}_1 \mathbf{3}_2$ and $\overline{\mathbf{3}}_2 \mathbf{3}_1$, while leaving the associated two pairs of Higgs(ino) doublets, $(\overline{\mathbf{2}}_1 + \mathbf{2}_1 + \overline{\mathbf{2}}_2 + \mathbf{2}_2)$ light. With the additional term $M_2(T_2)^2$ one of the pairs of doublets, $\overline{\mathbf{2}}_2 + \mathbf{2}_2$, can be made superheavy, thus leaving the correct spectrum for the MSSM and preserving the correct prediction of $\sin^2 \theta_W$. As was emphasized in Refs. [8,9], Higgsino-mediated proton decay,

which is a general problem for SUSY GUTs, can readily be suppressed in this scheme to acceptable levels by making M_2 only slightly smaller than the VEV of A^{ab} .

In previous papers [8,9] we showed that it was possible to construct a realistic and natural supersymmetric $SO(10)$ model with no fine-tuning using the Dimopoulos-Wilczek mechanism. It was found necessary in these papers to introduce the following Higgs representations to do the breaking of $SO(10)$: **54**, three **45**'s, **16**, and $\overline{\mathbf{16}}$. The **54** was required to give the DW form to the VEV of one of the adjoints. The pair of spinors was needed to break $SO(10)$ all the way to the standard model gauge group. And the presence of three adjoints was required to link the two sectors (the **54** + **45** to the $\overline{\mathbf{16}}$ + **16**) in such a way as to avoid goldstone bosons and preserve the DW form of the adjoint VEV. This will be explained further in Section 2.

An important question is whether the Higgs spectrum needed for the symmetry breaking as well as for achieving a natural doublet-triplet splitting can arise from an underlying superstring theory. It has been known for some time that conventional GUTs such as $SO(10)$ with scalars in the adjoint representation can indeed arise in the free fermionic formulation of superstrings [10]. Such a string construction requires the Kac-Moody level to be two or higher. In a recent paper, Chaudhury, Chung and Lykken [11] have given an explicit level two string construction of SUSY $SO(10)$ which has a single adjoint (along with arbitrary number of $\overline{\mathbf{16}}$, **16** and **10**) that survive below the Planck scale. Furthermore, these authors have classified the allowed representations that can emerge as massless chiral multiplets below the Planck scale at the level two construction: the number of adjoints is 0,

1 or 2, while the number of **54** is 0 or 1. No examples with more than one adjoint and/or one **54** have so far been constructed.

The Higgs spectrum used for a natural doublet–triplet splitting in Ref. [8,9], as it employs three adjoints, is not compatible with the superstring construction of Ref. [11]. In this paper, we therefore wish to address whether a realistic and natural SUSY $SO(10)$ model can be constructed with only a single adjoint Higgs. In particular, we wish to do away with the **54**, so that the spectrum will be identical to the explicit superstring construction of Ref. [11]. From a group theory point of view, a single adjoint and a $\overline{\mathbf{16}} + \mathbf{16}$ is sufficient to do the gauge symmetry breaking. We will show that it would also suffice to achieve a natural doublet–triplet splitting via the DW mechanism. In fact the Higgs content of the model we shall construct is minimal: there are two **10**’s and a **45** as required for the Dimopoulos-Wilczek mechanism, a $\overline{\mathbf{16}} + \mathbf{16}$ pair to complete the $SO(10)$ breaking, and a few singlets. As will be seen there is some flexibility in the details of the model. However, certain features are generic. In particular since the symmetry breaking mechanism relies on non-renormalizable operators, it turns out that there must be certain multiplets of colored Higgs(inos) which are very light compared to the GUT scale. As a result, the unification scale M_U is pushed above the minimal SUSY- $SU(5)$ value to about $2 \times 10^{17} \text{ GeV}$. This may be a welcome feature since M_U is now closer to the string compactification scale. These scalars also have an effect on $\sin^2 \theta_W$ which is predicted in the model to be somewhat larger than the SUSY- $SU(5)$ value. Though threshold corrections turn out to be surprisingly small, they can be large enough to somewhat compensate for the effects of these light colored fields, thus making $\sin^2 \theta_W$ consistent with

experiments. Nevertheless, one expects that $\sin^2 \theta_W$ will be somewhat on the large side in SUSY $SO(10)$ if the particle content is as currently suggested by superstring theory.

II. The Model

A. Achieving the DW form without a **54**.

In Refs. 8 and 9 the required Dimopoulos-Wilczek form of the adjoint vacuum expectation value was achieved by means of versions of the following superpotential, which we called the Srednicki sector [12].

$$W = m_1 A^2 + \lambda_1 S A^2 + m_2 S^2 + \lambda_2 S^3. \quad (1)$$

Here A is an adjoint and S is a **54**. If it is assumed that $\langle A \rangle$ is of the form $\text{diag}(b, b, a, a, a) \otimes (i\tau_2)$, then the $F_A = 0$ equation gives two conditions,

$$\begin{aligned} (m_1 - \frac{3}{2}s)b &= 0, \\ (m_1 + s)a &= 0, \end{aligned} \quad (2)$$

where the VEV of S is $\langle S \rangle = \text{diag}(-\frac{3}{2}s, -\frac{3}{2}s, s, s, s) \otimes I$. Clearly, one solution of this is $b = 0$ and $a \neq 0$ which is the DW form with $s = -m_1$.

If there is no **54** in the model, and the only tensors are one or more adjoints, then it appears that there is only one way to achieve the DW form, and that involves higher-dimension operators. In particular, with only a single adjoint one may write down

$$W = m(\text{tr} A^2) + \beta \frac{1}{M_{Pl}} (\text{tr} A^2)^2 + \beta' \frac{1}{M_{Pl}} (\text{tr} A^4). \quad (3)$$

This is easily seen to have as a solution $b = 0$ and $a = \sqrt{m M_{Pl} / (12\beta + 2\beta')}$. Notice that if the VEV of A is to be of order the GUT scale, M_G , then the

mass parameter of A has to be of order M_G^2/M_{Pl} . To explain the appearance of the higher-dimension operators it shall be assumed here and in what follows that all operators not forbidden by local symmetry will be induced by Planck-scale physics suppressed only by the dimensionally appropriate powers of the Planck mass, $M_{Pl} = 1.2 \times 10^{19}$ GeV.² This is what is generally expected in string theory.

B. Breaking $SO(10)$ to the Standard Model Group.

The VEV of the adjoint Higgs having the DW form breaks $SO(10)$ down to $SU(3)_c \times U(1) \times SO(4)$. To complete the breaking to the Standard Model group requires a $\overline{\mathbf{16}} + \mathbf{16}$, which will be denoted by $\overline{C} + C$. What is required is that the components of C and \overline{C} that are singlets under the Standard Model group acquire VEVs of order M_G . This is achievable in a number of ways. For example, there could be a singlet superfield, X , with coupling $X(\overline{C}C - M^2)$. Another possibility, similar to the adjoint sector, is to choose

$$W = m\overline{C}C + \beta \frac{1}{M_{Pl}}(\overline{C}C)^2 + \beta' \frac{1}{M_{Pl}}(\overline{C}\gamma^{ab}C)^2. \quad (4)$$

This is the most general form the superpotential for C and \overline{C} can take up to fourth order. (Terms such as C^4 and \overline{C}^4 do not play a role in symmetry breaking.)

C. The problem of linking the adjoint and spinor sectors.

There are, as just seen, two sectors of Higgs fields necessary to do the

²The proper expansion parameter in superstring theories may turn out to be the “reduced Planck mass” $\simeq 2 \times 10^{18}$ GeV rather than M_{Pl} . This variance can be readily accommodated into our analysis by correspondingly reducing the coefficients of the non-renormalizable operators (which are treated as free parameters).

breaking of $SO(10)$, one containing the adjoint which gets a VEV in $(B - L)$ direction (the Dimopoulos-Wilczek form), and a sector containing the pair of spinor fields which is needed to break the rank of the group to four and complete the breaking to the Standard Model group. A crucial problem in $SO(10)$ is how to link these sectors. They must certainly be linked, for otherwise there is nothing in the superpotential to determine the relative orientation of the VEVs in the two sectors. The result would be the existence of very light pseudogoldstone particles corresponding to those generators that are broken in both sectors. These are easily found to be in the representations $[(\mathbf{3}, \mathbf{2}, \frac{1}{6}) + (\mathbf{3}, \mathbf{1}, \frac{2}{3}) + \text{h.c.}]$ of the Standard Model group.

The reason that this is a problem is that terms that couple the two sectors will, in most cases, destabilize the Dimopoulos-Wilczek form of the adjoint VEV. For example, a $\overline{C}\gamma^{ab}CA^{ab}$ term leads to a linear term in b where $\langle A \rangle = \text{diag}(b, b, a, a, a) \otimes (i\tau_2)$.

The way this problem was solved in Refs. 8 and 9 was through a term of the form $\text{tr}(AA'A'')$. Here A is the adjoint which has a VEV in the DW form, and A'' is an adjoint which couples to $\overline{C}C$ and whose VEV thus does not have the DW form. A' is a third adjoint which is necessary because of the fact that $\text{tr}(AA'A'')$ is a totally antisymmetric term and therefore will vanish if any two of the adjoints are the same. Because of its total antisymmetry this term does not affect the VEVs of the fields. [This is easily seen from its contribution to F_A^{ab} , which is $(\langle A' \rangle \langle A'' \rangle)^{ba} = 0$, because the indices $[a, b]$ are antisymmetrized. It is assumed here that the VEVs of the adjoints are all in normal form: $(a_1, a_2, a_3, a_4, a_5) \otimes (i\tau_2)$.] However, this term does contribute to the masses of the would-be goldstone bosons. The drawback of this elegant

term is that it of necessity involves three distinct adjoints, while the present goal is to find a way to make do with only one or at most two adjoints [11].

It might be supposed that the same trick would work with the role of the A' and A'' being played by spinor-antispinor pairs contracted to form adjoints: $(\bar{C}\gamma^{ab}C)(\bar{C}'\gamma^{bc}C')A^{ca}/M_{Pl}^2$. While this term would indeed not destabilize the DW form of $\langle A \rangle$, that is not the case with other terms which can be obtained by contracting the same fields in different ways as, for example, $(\bar{C}\gamma^{ab}C)(\bar{C}'C')A^{ab}/M_{Pl}^2$. It is easily seen that there is no abelian symmetry which can allow one contraction of the fields while ruling out others. (With non-abelian discrete symmetries this can be done, but it is not clear that the necessary symmetries can emerge from string theory, and the examples we have found seem quite contrived, so we will not present them.)

In sum, there seems to be no way to link the two sectors without destabilizing the DW form, unless there are either three adjoints or non-abelian discrete symmetries that are respected by higher-dimension operators.

It seems that the only acceptable possibility is that the DW form is, indeed, destabilized, but only by a small enough amount that the gauge hierarchy is not destroyed. How small is small enough can be determined by examining the two-by-two matrices for the Higgs(ino) masses. These come from terms of the form $\lambda T_1^a A^{ab} T_2^b + M_2 (T_2)^2$. If the Higgs that couple to light quarks and leptons are in T_1 , then the Higgsino-mediated proton decay amplitude is proportional to $(\mathcal{M}^{-1})_{11} = M_2/(\lambda a)^2$ where \mathcal{M}^{-1} is the inverse of the two-by-two mass matrix of the colored Higgsinos. On the other hand, the μ -parameter of the light Higgs(ino) doublets in T_1 receive a contribution

$\delta\mu = (\lambda b)^2/M_2$. Thus

$$\delta\mu \cdot (\mathcal{M}^{-1})_{11} = (b/a)^2. \quad (5)$$

A comfortable agreement with the experimental limits on proton decay requires that $(\mathcal{M}^{-1})_{11} \lesssim (10^{17}\text{GeV})^{-1}$, while naturalness of the gauge hierarchy requires that $\delta\mu \lesssim 1 \text{ TeV}$. Thus, one requires that

$$b/a \lesssim 10^{-7}. \quad (6)$$

Such a small VEV for b can be achieved if the terms that destabilize the DW form by providing a linear term in b are high order and thus suppressed by powers of $1/M_{Pl}$. The price that is paid for this is that the masses of the pseudogoldstone bosons that arise from the same higher-dimension terms will be also very small compared to the GUT scale, as will be seen. This will be reflected in $\sin^2 \theta_W$ as well as in the unification scale.

D. A model of the $SO(10)$ -breaking sector.

In order that lower order terms that would disrupt the DW form and destroy the gauge hierarchy not be present it is necessary that there be symmetries. In the illustrative model of an $SO(10)$ -breaking sector now to be presented the symmetry is a $Z_4 \times Z_4$. The relevant fields are an adjoint, A , a spinor-antispinor pair, $\overline{C} + C$, and two pairs of singlets, (P_1, \overline{P}_1) , and (P_2, \overline{P}_2) . Under the first Z_4 , $A \rightarrow iA$, $P_1 \rightarrow iP_1$, $\overline{P}_1 \rightarrow -i\overline{P}_1$, and the other fields transform trivially. Under the second Z_4 , which will be denoted Z'_4 , $\overline{C} \rightarrow i\overline{C}$, $C \rightarrow iC$, $P_2 \rightarrow iP_2$, $\overline{P}_2 \rightarrow -i\overline{P}_2$, and the other fields transform trivially.

Under these symmetries the most general superpotential (up to the rele-

vant orders in M_{Pl}^{-1}) is given by

$$\begin{aligned}
W = & \frac{1}{M_{Pl}}(\alpha_1 P_1^2 + \overline{\alpha}_1 \overline{P}_1^2)tr(A^2) + \frac{\beta_1}{M_{Pl}}(tr(A^2))^2 + \frac{\beta'_1}{M_{Pl}}tr(A^4) \\
& + m_1 P_1 \overline{P}_1 + \frac{1}{M_{Pl}}(\gamma_1 P_1^4 + \overline{\gamma}_1 \overline{P}_1^4 + \gamma'_1 P_1^2 \overline{P}_1^2) \\
& + \frac{1}{M_{Pl}}(\alpha_2 P_2^2 + \overline{\alpha}_2 \overline{P}_2^2)\overline{C}C + \frac{\beta_2}{M_{Pl}}(\overline{C}C)^2 + \frac{\beta'_2}{M_{Pl}}(\overline{C}\gamma^{ab}C)^2 \\
& + m_2 P_2 \overline{P}_2 + \frac{1}{M_{Pl}}(\gamma_2 P_2^4 + \overline{\gamma}_2 \overline{P}_2^4 + \gamma'_2 P_2^2 \overline{P}_2^2) \\
& + \sum_i \frac{\delta_i}{M_{Pl}^5} \{ (\overline{C}C)^2 A^4 + (\overline{C}C P_2^2) A^4 + (\overline{C}C)^2 (A^2 P_1^2) + (\overline{C}C P_2^2) (A^2 P_1^2) \}_i.
\end{aligned} \tag{7}$$

Each term in the curly brackets on the right-hand side of Eq. (7) actually corresponds to several terms contracted in different ways. For example, there are seven distinct ways to contract $(\overline{C}C)^2 A^2$: $(\overline{C}\gamma^a \overline{C})(C\gamma^a C)tr(A^2)$, $(\overline{C}\gamma^a \overline{C})(C\gamma^b C)A^{ac}A^{cb}$, $(\overline{C}\gamma^a \overline{C})(C\gamma^{abcde}C)A^{bc}A^{de}$, $(\overline{C}\gamma^{abcde} \overline{C})(C\gamma^a C)A^{bc}A^{de}$, $(\overline{C}\gamma^{abcde} \overline{C})(C\gamma^{abcde}C)tr(A^2)$, $(\overline{C}\gamma^{abcde} \overline{C})(C\gamma^{abcfg}C)(A^{de}A^{fg} \text{ or } A^{df}A^{eg})$. Moreover, in each term in the curly brackets P_j^2 can be replaced by \overline{P}_j^2 . The many terms in the curly brackets are distinguished by the index ‘ i ’, and each has a distinct coefficient ‘ δ_i ’.

All the dimensionless coefficients in this superpotential, $\alpha_1, \beta_1, \dots, \delta_i$, are assumed to be of order unity. If the dimensionful parameters m_1 and m_2 are assumed to be of order M_G^2/M_{Pl} , then all the VEVs (except b) are of order M_G .

Defining $\langle \overline{C} \rangle = \langle C \rangle \equiv c$, then $\langle P_2 \rangle \sim \langle \overline{P}_2 \rangle \sim c$, and c sets the scale of $SO(10)$ breaking to $SU(5)$. Recalling that $\langle A \rangle \equiv \text{diag}(b, b, a, a, a) \otimes (i\tau_2)$, then $\langle P_1 \rangle \sim \langle \overline{P}_1 \rangle \sim a$, and a sets the scale of $SU(5)$ breaking to the Standard Model group.

The VEV b is determined by the $F_A = 0$ equation, which gives

$$b/a \approx (\delta_{eff}/\beta'_1) \left(\frac{c}{M_{Pl}} \right)^4. \quad (8)$$

Here δ_{eff} is some linear combination of all the δ_i that appear in the superpotential (Eq. (7)). If b/a is to be less than or of order 10^{-7} then c/M_{Pl} must be less than or of order 2×10^{-2} , which, as shall be seen later from solving the renormalization group equations for M_G , is reasonable.

This shows the importance of suppressing by discrete symmetry lower dimension operators linking the adjoint and spinor Higgs fields. If, for example, a term $\overline{C}CA$ were allowed, it would need to have a coefficient of order 10^{-9} to make $b/a \lesssim 10^{-7}$, which is a fine-tuning. By having the first operator which destabilizes the DW form of $\langle A \rangle$ be $O(1/M_{Pl}^5)$, it is possible to have a realistic model with all dimensionless parameters being of order unity.

E. The doublet-triplet splitting.

For reasons that will become apparent below, assume that the Higgs(ino) masses come from higher dimension operators.

$$W(T_1, T_2) = \lambda T_1^a A^{ab} T_2^b \left(\frac{P}{M_{Pl}} \right)^n + \lambda' (T_2)^2 \left(\frac{Q^{2n+1}}{M_{Pl}^{2n}} \right). \quad (9)$$

Then the doublet and triplet Higgsino mass matrices are given by

$$T_1 M T_2 = (\overline{\mathbf{2}}_1, \overline{\mathbf{2}}_2) \begin{pmatrix} 0 & i\Lambda b \\ -i\Lambda b & M_2 \end{pmatrix} \begin{pmatrix} \mathbf{2}_1 \\ \mathbf{2}_2 \end{pmatrix} + (\overline{\mathbf{3}}_1, \overline{\mathbf{3}}_2) \begin{pmatrix} 0 & i\Lambda a \\ -i\Lambda a & M_2 \end{pmatrix} \begin{pmatrix} \mathbf{3}_1 \\ \mathbf{3}_2 \end{pmatrix}, \quad (10)$$

where $M_2 = \lambda' (\langle Q \rangle^{2n+1} / M_{Pl}^{2n})$ and $\Lambda = \lambda (\langle P \rangle / M_{Pl})^n$. Thus the contribution

to the μ parameter is

$$\begin{aligned}
\delta\mu &= \Lambda^2 b^2 / M_2 = \left(\frac{\lambda^2}{\lambda'} \right) \frac{b^2 \langle P \rangle^{2n}}{\langle Q \rangle^{2n+1}} \\
&\sim b^2 / M_G \sim 10^{-14} a^2 / M_G \sim 10^{-14} M_G \\
&\sim 10^3 \text{ GeV},
\end{aligned} \tag{11}$$

where it has been assumed that the VEVs of P and Q are, like all the other VEVs, of order M_G , and the dimensionless couplings λ and λ' , like all the others, are of order unity.

The parameter that controls Higgsino-mediated proton decay, $(\mathcal{M}^{-1})_{11}$ is given by

$$(\mathcal{M}^{-1})_{11} = M_2 / (\Lambda^2 a^2) = \left(\frac{\lambda'}{\lambda^2} \right) \frac{\langle Q \rangle^{2n+1}}{a^2 \langle P \rangle^{2n}} \sim M_G^{-1}. \tag{12}$$

This is the correct order if proton decay is to be suppressed to realistic levels. Note that Eqs. (5) and (6) are satisfied.

The integer n appearing in the powers of M_{Pl} in Eq. (9) is determined by the following consideration. Any symmetry that allows the terms in Eq. (9) will also allow a term of the form $(T_1)^2 A^2 P^{2n} (\bar{Q})^{2n+1} / M_{Pl}^{4n+2}$, assuming there is a chiral superfield \bar{Q} with the opposite quantum numbers to Q . Since it has been assumed throughout that Planck-scale physics induces, unsuppressed except by powers of M_{Pl} , all higher-dimension operators allowed by local symmetry, it must be assumed that this operator exists also in the effective sub-Planck-scale theory. This would give a contribution to the μ parameter of order $M_G (M_G / M_{Pl})^{4n+2}$. It will be seen later that $M_G / M_{Pl} \approx \frac{1}{45}$, so that to avoid destroying the gauge hierarchy n must be ≥ 2 . One can impose a local $U(1)$ symmetry that guarantees the form of Eq. (9) with no lower

dimension operators contribute to μ . For example, if $n = 2$, the $U(1)$ charges of (P, Q, T_1, T_2) can be chosen to be $(2, -2, 5, 9)$ with all the remaining fields having zero charge. Instead of a $U(1)$ symmetry it is possible to use a discrete subgroup of the $U(1)$ (eg: Z_9 or Z_{18} in the above example). As shown in Ref. 9, it is straightforward to make these symmetries free of anomalies so that they are “local”.

F. The spectrum of the model.

Of the 45 gauge bosons of $SO(10)$, 12 are the gauge bosons of the Standard Model and remain light. The rest have masses of order M_G . In particular, $M^2((\mathbf{3}, \mathbf{1}, \frac{2}{3}) + h.c.) = 4g^2(c^2 + a^2)$, $M^2((\mathbf{3}, \mathbf{2}, \frac{1}{6}) + h.c.) = g^2(4c^2 + a^2)$, $M^2((\mathbf{1}, \mathbf{1}, \pm 1)) = M^2((\mathbf{1}, \mathbf{1}, \mathbf{0})) = 4g^2c^2$, and $M^2((\mathbf{3}, \mathbf{2}, -\frac{5}{6}) + h.c.) = g^2a^2$.

Of the 77 ($= \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}}$) Higgs(ino) components involved in the breaking of $SO(10)$ to the Standard Model, 33 are eaten to give the massive gauge multiplets just enumerated. 11 components of the $\mathbf{16} + \overline{\mathbf{16}}$ (namely $\overline{\mathbf{5}} + \mathbf{5} + \mathbf{1}$ under $SU(5)$) acquire mass of order $c^2/M_{Pl} \sim m_2$. These fields have masses that are very nearly $SU(5)$ invariant because their coupling to $\langle A \rangle$ is so weak (ie. $O(M_G^6/M_{Pl}^5)$). They therefore have a negligible effect on $\sin^2 \theta_W$.

Further, 15 components of the adjoint, A , acquire masses of order $a^2/M_{Pl} \sim m_1$. In particular, $M((\mathbf{1}, \mathbf{1}, \mathbf{0})) = M((\mathbf{8}, \mathbf{1}, \mathbf{0})) = 4\beta'_1 a^2/M_{Pl}$, $M((\mathbf{1}, \mathbf{3}, \mathbf{0})) = M((\mathbf{1}, \mathbf{1}, \pm 1)) = M((\mathbf{1}, \mathbf{1}, \mathbf{0})) = 2\beta'_1 a^2/M_{Pl}$.

Finally, there are 18 pseudogoldstone bosons (and their fermionic partners) that come from both A and $\overline{C} + C$. Their masses are $M((\mathbf{3}, \mathbf{1}, \frac{2}{3}) + h.c.) = \delta_{eff}^{(1)} a^2 c^2 (c^2 + a^2)/M_{Pl}^5$, and $M((\mathbf{3}, \mathbf{2}, \frac{1}{6}) + h.c.) = \delta_{eff}^{(2)} a^2 c^2 (4c^2 + a^2)/M_{Pl}^5$, where $\delta_{eff}^{(1)}$ and $\delta_{eff}^{(2)}$ are some linear combinations of the δ_i appearing in the superpotential. With $M_G/M_{Pl} \sim \frac{1}{45}$ (as shall be found later)

these pseudogoldstones have masses of order 2×10^9 GeV.

In the **10**'s, T_1 and T_2 , there are two $[(\mathbf{3}, \mathbf{1}, -\frac{1}{3}) + h.c.]$ pairs, the product of whose masses is seen from Eq. (10) to be $\Lambda^2 a^2 \sim (M_G^3/M_{Pl}^2)^2$. There is one pair of $(\mathbf{1}, \mathbf{2}, -\frac{1}{2}) + h.c.$ with mass M_2 , and one light pair with mass of order the weak scale.

G. Realistic Fermion masses.

The three families of quarks and leptons belonging to **16** of $SO(10)$ (denoted by F_I , $I=1-3$) have the following Yukawa couplings to T_1 and \overline{C} :

$$\lambda_{IJ} F_I F_J T_1 + \lambda'_{IJ} F_I F_J (\overline{C} \cdot \overline{C} N / M_{Pl}^2) , \quad (13)$$

where N is a gauge singlet. Clearly such terms respect all the symmetries of the model discussed earlier. The coupling to T_1 gives rise to the Dirac masses of all fermions, while the coupling to \overline{C} results in heavy Majorana neutrino masses for ν_R 's. In order to correct the bad $SU(5)$ mass relations, it is necessary for the light quark and lepton masses to depend on the breaking of $SU(5)$. Therefore they must couple to A . Of course, the direct coupling of A to $F_I F_J$ is not allowed by $SO(10)$. One idea that has been suggested in the literature [13,9] is that there are additional vector-like representations of quarks and leptons. If, for example, there is a $\mathbf{16} + \overline{\mathbf{16}}$ (denoted by $F + \overline{F}$), then A may couple as follows:

$$\lambda_I F_I \overline{F} \left(\frac{AN}{M_{Pl}} \right) + m F \overline{F} . \quad (14)$$

This allows realistic quark and lepton mass relations [13,9]. Note that the gauge hierarchy is unaffected by these vector fermions.

H. Cosmology of the pseudogoldstone bosons.

As noted in II.F, the model has 18 pseudogoldstone bosons (and their fermion partners) belonging to $[(\mathbf{3}, \mathbf{1}, \frac{2}{3}) + (\mathbf{3}, \mathbf{2}, \frac{1}{6}) + h.c.]$ under the Standard Model gauge group. They have masses of order $2 \times 10^9 \text{ GeV}$. Since all the gauge bosons with which they interact have masses of order $M_U \sim 2 \times 10^{17} \text{ GeV}$, it is important to check if these pseudogoldstones are so long-lived as to cause problems for cosmology.

The $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ (denoted by χ_1) pseudogoldstone can readily decay into light fermions using the $T_1^a A^{ab} T_2^b$ interaction of Eq. (9). χ_1 decays into a light doublet from T_1 and a heavy (virtual) color triplet from T_2 . By using the same $T_1 A T_2$ vertex, the color triplet in T_2 converts into a color triplet in T_1 , which has Yukawa couplings to the light quarks and leptons. The amplitude for this decay $\chi_1 \rightarrow H_1 F_I F_J$ goes as $1/M_G$ with the decay rate $\Gamma_d \sim m_{\chi_1}^3/M_G^2 \sim 10^{-6} \text{ GeV}$. Comparing Γ_d with the expansion rate of the universe $\Gamma_{exp} \sim T^2/M_{Pl}$, we see that the freeze-out temperature is $T_* \sim 10^7 \text{ GeV}$, which is sufficiently high and quite safe.

As for the $(\mathbf{3}, \mathbf{1}, \mathbf{2}/\mathbf{3})$ pseudogoldstone (denoted by χ_2), there is no direct coupling with the light Higgs in T_1 . However, it also decays quite fast. The interaction of A listed in Eq. (14), along with the superpotential terms $\alpha A^2 P^2$ and βA^4 (see Eq. (7)) lead to the D -term

$$\lambda_I \lambda_J \overline{F_I} \partial_\mu \gamma^\mu F_J A \left(\frac{1}{16\pi^2} \frac{\langle A \rangle \alpha \beta}{M_G^2} \right), \quad (15)$$

which arises through a one-loop diagram. The decay rate for $\chi_2 \rightarrow F_I F_J$ is then $\Gamma_d \sim \left(\frac{1}{16\pi^2} \frac{\lambda_I \lambda_J \alpha \beta}{M_G} \right)^2 m_{\chi_2}^3 \sim (\lambda_I \lambda_J \alpha \beta)^2 \times 10^{-11} \text{ GeV}$. The corresponding freeze-out Temperature is $T_* \sim (\lambda_I \lambda_J \alpha \beta) \times 10^4 \text{ GeV}$, which is also quite

safe.

It is easy to verify that these pseudogoldstone bosons do not mediate proton decay at an unacceptable level. The effective interaction of Eq. (15) leads to a proton decay amplitude proportional to the light quark masses (as in the usual dimension 5 proton decay of SUSY-GUT) and a factor $1/(16\pi^2 M_G)$. This rate is negligibly small. Similarly, box diagrams with internal particle being F, \bar{F} can be seen to have an amplitude $\sim 1/M_{Pl}^2$ with the usual light quark Yukawa suppression factors, which is also small.

III. The calculation of $\sin^2 \theta_W$

For a number of reasons the uncertainties in $\sin^2 \theta_W$ due to physics at large scales are relatively quite small in this model compared to what one might expect in $SO(10)$. First, the string-theory-motivated constraints that have been imposed have meant that there is only one large representation of Higgs(inos), namely the adjoint. Second, because of the extremely weak coupling of the spinor-antispinor pair of Higgs(inos) to the adjoint, they contribute negligibly to $\sin^2 \theta_W$ as already noted, since they are almost perfectly $SU(5)$ degenerate. And, third, the gravitational contributions are small since the only possible term of order M_{Pl}^{-1} , namely $\text{tr} \frac{\langle A \rangle}{M_{Pl}} F_{\mu\nu} F^{\mu\nu}$ vanishes because of the antisymmetry of A . The terms of order M_{Pl}^{-2} will produce an uncertainty, at most, of order 10^{-3} in $\sin^2 \theta_W$.

Moreover, the presence of the pseudogoldstones at intermediate scales has the effect of somewhat pushing up $\sin^2 \theta_W$. One expects, therefore, that $\sin^2 \theta_W$ will lie at the high end of the presently allowed range. This will be

quantified shortly.

The $SU(5)$ gauge bosons that mediate proton decay (the $(\mathbf{3}, \mathbf{2}, -\frac{5}{6}) + h.c.$) have mass ga which will be defined to be M_G . The breaking of $SO(10)$ to $SU(5)$ contributes to the mass-squared of the gauge bosons an amount $(2gc)^2$ which will be denoted M_{10}^2 . It will be convenient to define $x \equiv M_{10}/M_G = 2c/a$.

From the one-loop renormalization group equations and using the spectrum of particles listed in Section 2, we arrive at

$$\begin{aligned} \ln\left(\frac{M_U}{M_{Pl}}\right) &= \frac{\pi}{17\alpha} - \frac{8\pi}{51\alpha_3} - \frac{1}{17}\ln(\rho_1^2\rho_5) + \frac{10}{17}\ln\left(\frac{M_Z}{M_{Pl}}\right) \\ \alpha_G^{-1} &= \frac{7}{17}\alpha^{-1} - \frac{5}{51}\alpha_3^{-1} - \frac{48}{17\pi}\ln\rho_1 + \frac{37}{34\pi}\ln\rho_5 + \frac{89}{34\pi}\ln\left(\frac{M_Z}{M_{Pl}}\right) \\ \sin^2\theta_W(M_Z) &= \frac{5}{34} + \frac{31}{51}\frac{\alpha}{\alpha_3} + \frac{5\alpha}{34\pi}\ln\left(\frac{\rho_1}{\rho_5}\right) + \frac{\alpha}{17\pi}\ln\rho_1 - \frac{9\alpha}{17\pi}\ln\left(\frac{M_Z}{M_{Pl}}\right) \end{aligned} \quad (16)$$

Here $\rho_1 = (2\beta'_1/g^2)(M_G/M_U)^2$ is the coefficient of the Higgs(ino) mass from the **45** which is of order M_G^2/M_{Pl} , and $\rho_5 = \delta_{eff}^{(1)}\frac{x^2}{4}(1 + \frac{x^2}{4})(\frac{M_G}{gM_U})^6$ is the coefficient of the Higgs(ino) pseudogoldstone multiplet which has a mass of order M_G^6/M_{Pl}^5 . In the above, we have ignored the mass-splitting between various multiplets that are of the same order. This will be treated as part of the threshold corrections. $M_G = ga$ is the mass of the $SU(5)$ gauge boson and M_U is the scale at which the three couplings unify.

Using $\alpha_3(M_Z) = 0.12$ and $\alpha(M_Z) = 1/127.9$ as inputs, we see that $\alpha_G^{-1} = 19$, or $g = 0.81$ corresponding to $\rho_1 = \rho_5 = 1$. The unification scale is found to be

$$M_U/M_{Pl} \cong (2.3 \times 10^{-2})[x^2(1 + \frac{x^2}{4})]^{-\frac{1}{17}}. \quad (17)$$

For x of order unity, $M_U/M_{Pl} \simeq 1/45$. The one-loop prediction for $\sin^2\theta_W$,

ignoring threshold effects for now, is

$$\sin^2\theta_W|_{1-loop} = 0.2384 - \frac{5\alpha}{34\pi} \ln\left(\frac{\delta_{eff}^{(1)} x^2}{8g^4\beta'_1} \left(1 + \frac{x^2}{4}\right)\right) + \frac{\alpha}{17\pi} \ln\left(\frac{2\beta'_1}{g^2}\right) - \frac{8\alpha}{17\pi} \ln\left(\frac{M_G}{M_U}\right). \quad (18)$$

The logarithmic terms reflect the contribution of the light pseudogoldstones. One can estimate the ratio $\delta_{eff}^{(1)}/\beta'_1$ by considering the ratio of VEVs b/a . From Eqs. (6) and (8) one has that $\delta_{eff}/\beta'_1 \lesssim 10^{-7} \left(\frac{x}{2g} \frac{M_G}{M_{Pl}}\right)^{-4}$. This gives $\ln(\delta_{eff}/\beta'_1) \lesssim -\ln(\frac{x^4}{4})$. Thus the second term in Eq. (18) tends to be a *positive* contribution to $\sin^2\theta_W$. The δ_{eff} appearing in the expression for b/a is not the same linear combination of the δ_i that appears in the pseudogoldstone masses and that has been denoted $\delta_{eff}^{(1)}$. However, if all the δ_i are assumed to be comparable, the difference as far as $\sin^2\theta_W$ is concerned should be negligible.

Let us now turn to the two-loop and threshold corrections to $\sin^2\theta_W$ in the model. The two-loop correction to $\sin^2\theta_W$ (including a conversion factor to go from \overline{MS} to \overline{DR} scheme) is obtained numerically to be

$$\sin^2\theta_W|_{2-loop} = +0.0037. \quad (19)$$

The correction arising from the splitting among the superheavy gauge multiplets is

$$\Delta \sin^2\theta_W|_{gauge} = -\frac{\alpha}{10\pi} [6 \ln x + 15 \ln(4 + x^2)^{\frac{1}{2}} - 21 \ln(1 + x^2)^{\frac{1}{2}}]. \quad (20)$$

The correction from the splittings among Higgs(ino) multiplets with masses of order a^2/M_{Pl} is

$$\Delta \sin^2\theta_W|_{Higgs} = +\frac{\alpha}{30\pi} (21 \ln 2). \quad (21)$$

The correction coming from the splitting between the pseudogoldstone multiplets is given by

$$\Delta \sin^2 \theta_W|_{pseudos} = \frac{\alpha}{30\pi}(-21) \left(\ln \left(\frac{1+x^2}{1+x^2/4} \right) + \ln \left(\frac{\delta_{eff}^{(2)}}{\delta_{eff}^{(1)}} \right) \right). \quad (22)$$

The correction, finally, from the splittings among the heavy fields in T_1 and T_2 is given by

$$\Delta \sin^2 \theta_W|_{T_i} = \frac{\alpha}{30\pi} 9 \ln \left(\frac{\Lambda^2 a^2}{M_2 M_G} \right) = -\frac{\alpha}{30\pi} 9 \ln(M_G(\mathcal{M}^{-1})_{11}). \quad (23)$$

Combining all the contributions one finds that

$$\begin{aligned} \sin^2 \theta_W(M_Z) &= \frac{5}{34} + \frac{31}{51} \frac{\alpha}{\alpha_3} - \frac{9\alpha}{17\pi} \ln \left(\frac{M_Z}{M_{Pl}} \right) - \frac{19\alpha}{170\pi} \ln 2 + \frac{18\alpha}{17\pi} \ln |g| + 0.0037 \\ &+ \frac{\alpha}{17\pi} \ln(2\beta'_1) - \frac{5\alpha}{34\pi} \ln \left(\frac{\delta_{eff}^{(1)} x^4}{2\beta'_1} \right) - \frac{231\alpha}{170\pi} \ln \left(\frac{M_G}{M_U} \right) \\ &- \frac{21\alpha}{30\pi} \ln \left(\frac{\delta_{eff}^{(1)}}{\delta_{eff}^{(2)}} \right) - \frac{3\alpha}{10\pi} \ln[M_G(\mathcal{M}^{-1})_{11}] \\ &+ \frac{\alpha}{60\pi} \left\{ 21 \ln(1+x^2) - \frac{201}{17} \ln(4+x^2) - \frac{156}{17} \ln x^2 \right\} \end{aligned} \quad (24)$$

The uncertainty in $\sin^2 \theta_W$ is $\pm 0.0033 \pm 0.0014 \pm 0.0006$ [2], where the first number corresponds to the experimental errors in $\alpha(M_Z)$ and $\alpha_3(M_Z)$, the second one to SUSY particle threshold and the last one to the top and higgs thresholds.

The terms in the first line of Eq. (24) is unambiguous and adds up to 0.2415. Unless there is some cancellation from the other terms, $\sin^2 \theta_W$ will be incompatible with experiments. The $\ln(2\beta'_1)$ term has an extremely small coefficient and is negligible, the second term on line 2 is, as noted earlier, tends to be a positive contribution (see Eq. (8)). The last term in line 2

is nearly zero or positive. As for the terms in the curly brackets in the last line, it is positive for $x < 0.5$, but it can be negative for larger x , with its minimum being -1.9×10^{-4} . The term with $\ln(M_G(\mathcal{M}^{-1})_{11})$ is probably positive (if the proton is not to decay too fast) and is of order 10^{-3} at most. That leaves us with the $\ln(\delta_{eff}^{(1)}/\delta_{eff}^{(2)})$ term to be the only term that can be significantly negative. In its absence, $\sin^2\theta_W$ would come out too large. But this term, depending on the unknown ratio of δ 's can bring $\sin^2\theta_W$ to agreement if the logarithm is about 3. It should be emphasized that the two δ 's are *independent* parameters of the model and can easily differ from each other by some factor of order unity.

IV. Conclusions

In this paper we have presented a very simple scheme for the gauge symmetry breaking in the context of SUSY- $SO(10)$ GUT. The Higgs system employed consisting of a single **45** along with a $\overline{\mathbf{16}} + \mathbf{16}$ and a pair of **10**, is the absolute minimum required for symmetry breaking and a natural doublet-triplet splitting without fine-tuning of parameters. Such a spectrum has been shown to arise in the free fermionic formulation of superstrings [11]. The mass of the light Higgs doublet is protected by local symmetry against higher dimensional operators induced by Planck scale physics to sufficiently high order. The model presented here is a simplification over earlier attempts along these lines [8,9,14].

Since the symmetry breaking mechanism relies on non-renormalizable operators (without such operators $SO(10)$ can only break down to $SU(5)$),

some of the Higgs(ino)s in the model turn out to have masses below the GUT scale. These pseudogoldstone multiplets affect $\sin^2\theta_W$ as well as the unification scale M_U . (In Refs. [8,9], it was required that the spectrum below the GUT scale should be the same as the MSSM spectrum, so $\sin^2\theta_W$ and M_U predictions were essentially the same as in minimal SUSY- $SU(5)$. The price to be paid was the necessity of having three adjoints.) M_U is found to be about $2 \times 10^{17} \text{ GeV}$, which is closer to the string compactification scale, while $\sin^2\theta_W$ is somewhat on the large end of the presently allowed range. Threshold corrections to $\sin^2\theta_W$ turn out to be quite small, but they are large enough to make the prediction consistent with experiments. We have also shown that these pseudogoldstone Higgs(ino)s do not pose any problem for cosmology, as they decay sufficiently fast in the early universe. They also do not mediate proton decay at an unacceptable level.

The simplicity of the Higgs sector of the model also means that all the couplings will remain perturbative in the momentum range from M_{GUT} to M_{Pl} . Realistic fermion masses including small neutrino masses can arise naturally within this scheme. In fact, the model may prove to be a fertile ground for implementing predictive schemes for quark and lepton masses. This will be the subject of a future investigation.

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